

Functions, their properties, and graphical analysis using modern mathematical software

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Abstract: This study examines the concept of functions, their fundamental properties, and methods of graphical analysis using modern mathematical software. The research focuses on integrating traditional analytical approaches with interactive digital visualization to improve the understanding of functional relationships. GeoGebra was used to construct and analyze graphs, investigate parameter changes, and examine important characteristics such as domain, range, zeros, extrema, monotonicity, and symmetry. The methodology combined algebraic expressions, numerical tables, graphical representations, and analytical verification. The results were evaluated through a comparison of pre-test and post-test performance between experimental and control groups. The experimental group demonstrated an increase in average performance from 55 to 82 points, while the control group increased from 54 to 60 points. These findings indicate that interactive graphical environments can support students in connecting symbolic and visual representations of functions. The study concludes that modern mathematical software can complement conventional teaching methods by providing dynamic visualization, facilitating mathematical exploration, and promoting a deeper conceptual understanding of functions and their graphical properties.

Keywords: functions, function properties, graphical analysis, GeoGebra, mathematical software, mathematics education.

INTRODUCTION

The concept of a function occupies a central position in mathematics and provides a fundamental framework for describing relationships between quantities and variables. Functions are widely used not only within algebra and mathematical analysis but also in physics, engineering, economics, computer science, and other scientific disciplines. A comprehensive understanding of functions requires students to recognize the relationship between their analytical expressions, numerical values, and graphical representations [6]. In particular, the ability to determine the domain and range, zeros, intervals of increase and decrease, extrema, continuity, symmetry, and other properties of a function is essential for developing mathematical reasoning and problem-solving skills.

Despite the fundamental role of functions in mathematics education, students frequently experience difficulties when interpreting and connecting different representations of functional relationships. A function can simultaneously be represented by an algebraic formula, a table of values, or a graph, and each representation emphasizes different aspects of the same mathematical object. Difficulties may arise when learners attempt to move from an equation to its graphical representation or interpret the meaning of a particular point, interval, or change in the graph [3]. Therefore, effective instruction should not be limited to the mechanical construction of graphs but should emphasize the interpretation of graphical features and their relationship to the corresponding algebraic properties.

The development of digital technologies has created new opportunities for improving the teaching and learning of functions. Modern mathematical software can transform abstract mathematical relationships into dynamic visual representations, allowing learners to observe changes in a graph as parameters or input values are modified. Such an approach can facilitate the exploration of functional behavior and make mathematical concepts more accessible through visual and interactive learning. Technology-supported mathematics education is particularly valuable when it is used not simply as a computational tool but as an environment for investigation, experimentation, and interpretation [10].

Dynamic mathematical applications such as GeoGebra and Desmos provide tools for constructing and analyzing function graphs in real time. With these applications, users can plot different types of functions, modify coefficients, compare several graphs, identify characteristic points, and investigate transformations such as translations, reflections, and changes in scale. For example, changing the coefficients of a quadratic function allows students to immediately observe corresponding changes in the position, direction, and shape of its graph [5]. This dynamic interaction can help establish a clearer connection between symbolic expressions and their graphical consequences.

The use of technology can also support the systematic investigation of the main properties of functions. Instead of considering domain, range, zeros, monotonicity, extrema, and symmetry as isolated theoretical concepts, learners can examine these characteristics directly on interactive graphs and relate them to analytical calculations. Consequently, graphical analysis becomes an active process in which students formulate hypotheses, test mathematical relationships, and verify their results. Such activities can contribute to the development of conceptual understanding rather than relying exclusively on memorization of rules and procedures [14].

At the same time, the effectiveness of educational software depends on how it is integrated into the learning process. Technology itself does not automatically guarantee improved mathematical understanding. Its educational value increases when digital tools are combined with appropriate mathematical tasks, analytical reasoning, teacher guidance, and interpretation of obtained results. Therefore, modern software should be regarded as a complementary instrument that supports traditional analytical methods while providing additional opportunities for visualization and experimentation [1].

In this context, the integration of classical methods for studying functions with modern digital technologies represents an important direction in contemporary mathematics education. The combination of analytical, numerical, and graphical approaches enables learners to examine functions from different perspectives and develop a more comprehensive understanding of their properties.

Accordingly, this study focuses on the concept of a function, the analysis of its fundamental properties, and methods for constructing and interpreting graphs using modern mathematical software [6]. Particular attention is given to the possibilities of interactive graphical environments for visualizing functional relationships and supporting a deeper understanding of mathematical concepts.

RESEARCH METHODOLOGY

The study employed a technology-supported methodological approach to investigate functions, their properties, and methods of graphical analysis using modern mathematical software. The research combined traditional analytical methods with interactive digital visualization, with particular emphasis on GeoGebra and Desmos.

The research procedure consisted of several stages. First, the basic concepts and properties of functions were reviewed, including domain, range, zeros, intercepts, monotonicity, extrema, and symmetry. Next, selected linear, quadratic, and exponential functions were entered into the software and their graphs were constructed. The parameters of the functions were then changed interactively to examine their influence on the shape and position of the graphs [8].

Students were encouraged to compare algebraic expressions, numerical tables, and graphical representations. The obtained graphical results were verified through analytical calculations. This approach allowed the relationship between algebraic parameters and graphical characteristics to be investigated more clearly and supported the development of students' conceptual understanding and mathematical reasoning.

Table 1. Main stages of the research

Stage	Activity	Result
1	Review of function concepts	Theoretical understanding
2	Construction of graphs	Visual representation
3	Interactive parameter changes	Investigation of transformations
4	Analysis of function properties	Identification of key characteristics
5	Analytical verification	Validation of graphical results

**Note: The table summarizes the principal methodological stages used to investigate functions and their graphical properties with modern mathematical software.*

The graphical analysis was performed using modern software tools that enable dynamic visualization and immediate observation of changes in functions.

$$y = f(x) \quad (1)$$

$$f(x) = a x^2 + b x + c \quad (2)$$

$$m = \frac{(f(x_2) - f(x_1))}{(x_2 - x_1)} \quad (3)$$

The mathematical analysis was based on three complementary approaches: the general concept of a function, the analysis of quadratic functions, and the determination of the average rate of change. These approaches were used to examine the relationships between input and output variables, investigate the influence of algebraic parameters on graphical characteristics, and determine the increasing or decreasing behavior of functions. Modern mathematical software, particularly GeoGebra, was used to visualize these relationships and compare analytical calculations with graphical representations. This combination of analytical and graphical methods provided a systematic framework for studying functions and their properties.

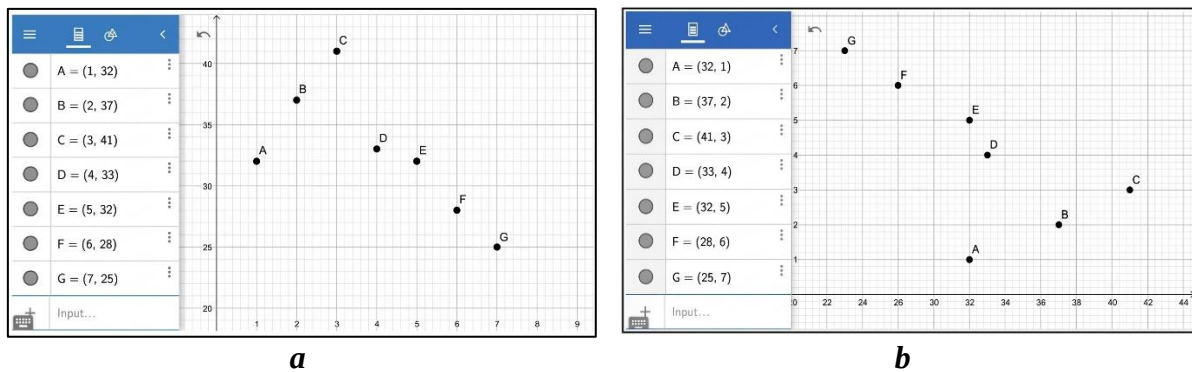


Figure 1. Graphical representation of input–output data using GeoGebra.

(a) Original input–output data (b) Interchanged input–output data.

(Source: Adapted from Soots and Shafer (2018)).

The results obtained through graphical visualization were compared with mathematical calculations to ensure their accuracy and to demonstrate the connection between analytical and graphical approaches. Thus, the methodology integrates mathematical analysis, graphical visualization, numerical representation, and digital technologies into a unified approach to studying functions.

RESULTS

The results of the study demonstrate the potential contribution of modern mathematical software to students' understanding of functions and their graphical representations. The analysis focused on students' performance before and after the implementation of GeoGebra-assisted learning activities. The obtained results were examined by comparing the average pre-test and post-test scores of the experimental and control groups.

The comparison showed a noticeable difference between the two groups. The experimental group, which participated in GeoGebra-assisted learning activities, increased its average score from 55 in the pre-test to 82 in the post-test, corresponding to an improvement of 27 percentage points. In

comparison, the control group increased from 54 to 60, representing an improvement of 6 percentage points.

Table 1. Comparison of Pre-test and Post-test Results

Group	Average Pre-test	Average Post-test	Improvement (%)
Experimental Group	55	82	27
Control Group	54	60	6

The results indicate that the experimental group demonstrated a considerably larger increase in average performance than the control group. At the beginning of the study, the groups had relatively similar pre-test scores (55 and 54). Following the instructional intervention, however, the experimental group achieved an average score of 82, while the control group reached an average of 60. This difference suggests that the use of interactive graphical activities provided additional opportunities for students to explore and interpret functional relationships.

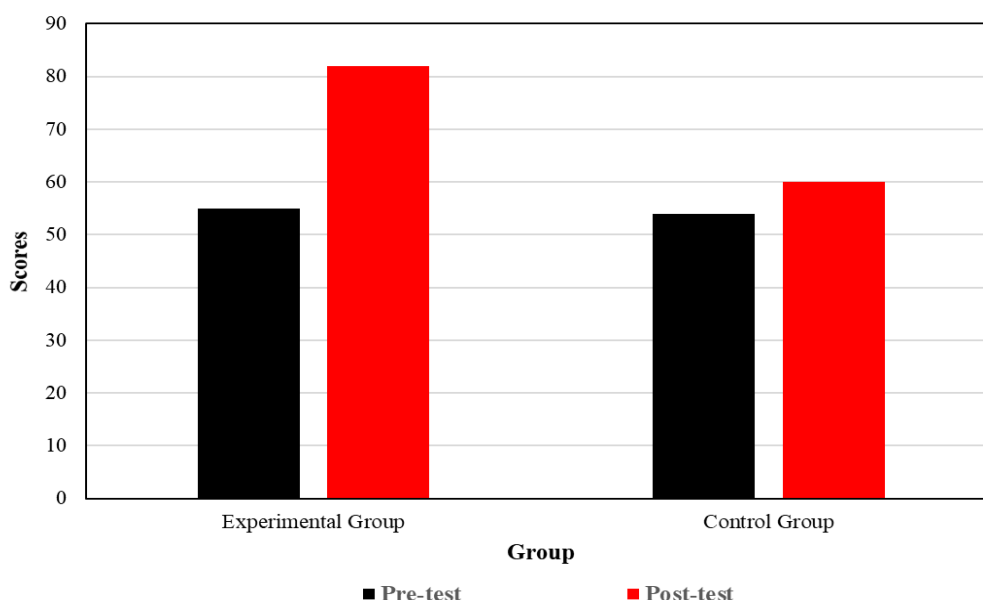


Figure 2. Comparison of pre-test and post-test scores between the experimental and control groups.

The graphical comparison further illustrates the changes in students' performance. The experimental group shows a substantial increase between the initial and final assessments, whereas the change in the control group is comparatively smaller. The results therefore demonstrate a positive association between the implementation of GeoGebra-assisted activities and students' performance in tasks related to functions and graphs.

In addition to the quantitative results, the graphical activities supported students in connecting different representations of mathematical functions. Through GeoGebra, students could simultaneously examine algebraic expressions, numerical values, and their corresponding graphs.

Changes in function parameters could also be observed dynamically, allowing students to identify relationships between algebraic coefficients and graphical characteristics.

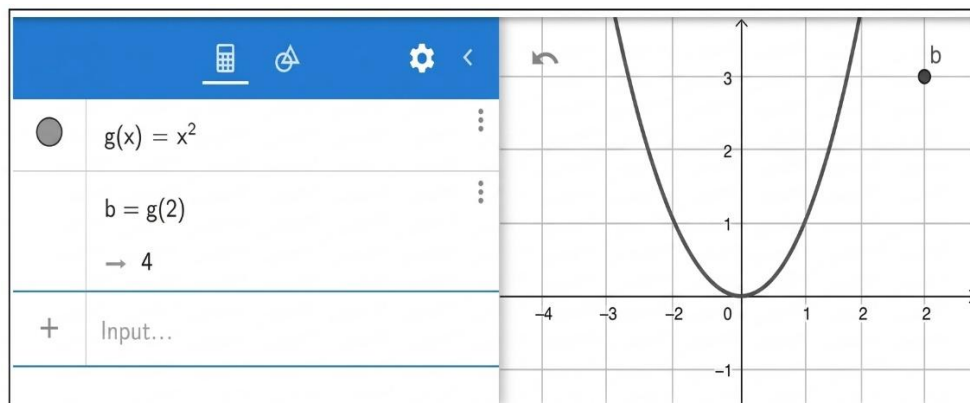


Figure 3. GeoGebra visualization of a quadratic function

Overall, the findings show that integrating GeoGebra into the study of functions was accompanied by a substantial improvement in the experimental group's assessment results. The average score increased by 27 points in the experimental group compared with 6 points in the control group. The results also indicate that interactive graphical visualization can complement traditional analytical methods by enabling students to observe, investigate, and verify functional relationships more directly.

It should be noted that the reported results describe the performance of the groups included in this study and should not automatically be generalized to all learners or educational settings. Nevertheless, the findings provide quantitative evidence supporting further investigation of interactive mathematical software as a component of function and graph instruction.

CONCLUSION

The study demonstrated that the integration of modern mathematical software into the study of functions provides an effective way to connect analytical, numerical, and graphical representations. The use of interactive tools such as GeoGebra enables learners to construct function graphs, examine their main properties, and observe how changes in algebraic parameters affect graphical behavior. The combination of theoretical analysis with dynamic visualization can therefore support a clearer understanding of domains, ranges, zeros, extrema, monotonicity, symmetry, and other characteristics of functions.

The comparison of pre-test and post-test results also showed an increase in the performance of the experimental group following the use of software-supported learning activities. Interactive graphical analysis encouraged learners to explore mathematical relationships, verify analytical calculations, and interpret graphical results more independently. Overall, the findings indicate that

modern digital tools can serve as a valuable complement to traditional methods of teaching functions and can contribute to a more interactive, visual, and conceptually oriented approach to mathematical learning.

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